ture of He^3. They also brought to light a slight discrepancy, concerning Fig. 1 of their paper, which is also reproduced in Ref. 1. The figure is incorrectly drawn, so that the He^3 and He^4 profiles overlap more than they should. The reason is that different scales are used in the two parts of the figure. Figure 3 shows the theoretical predictions of Fred *et al.*, correctly drawn, and below are shown the profiles of our He^3 and He^4 lamps also in their correct relative positions. The importance of this is in relating the optical pumping signal observed in the experiments to the polarization of the sample. Colgrove *et al.* assumed that both hyperfine components of the He^3 $2\,^3S_1-2\,^3P_0$ line were equally absorbent to He^4 light, and obtained the relation:

$$\frac{\Delta I}{I}=\frac{P(11-2P-P^2)}{(6+2P^2)}\simeq\frac{11P}{6}, \quad \text{if } P \text{ is small.}$$

In view of the above, this should be amended to

$$\frac{\Delta I}{I}=\frac{P(15-10P+3P^2)}{(6+2P^2)}\simeq\frac{5P}{2}, \quad \text{if } P \text{ is small.}$$

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K-Ionization Cross Sections for Relativistic Electrons

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Experimental values of the K-shell ionization cross section are determined from scintillation spectrometer measurements of the K x rays emitted when thin targets of tin and gold are bombarded by 50-, 100-, 200-, and 500-keV electrons. For these energies and atomic numbers, the experimental results show differences from the cross sections given by Burhop's nonrelativisitic calculations and by Perlman's relativistic calculations, but show good agreement with the predictions of Arthurs and Moiseiwitsch, who employed relativistic free-particle wave functions for the projectile electron and nonrelativistic wave functions for the atomic electron.

1. INTRODUCTION

THEOERTICAL studies of atomic K-shell ionization by electrons may be divided into two main groups. First are the nonrelativistic calculations by Bethe,[1] Massey and Mohr,[2] Soden,[3] Massey and Burhop,[4] Wetzel,[5] and Burhop,[6] and second are the relativistic calculations by Arthurs and Moiseiwitsch[7] and Perlman.[8] In most cases, these calculations are not in a simple analytical form and must be evaluated numerically. Burhop[6] has evaluated the K-ionization cross sections for the higher atomic numbers, 28 (nickel), 47 (silver), and 80 (mercury), and among the nonrelativistic calculations listed above, his results are most pertinent to the present investigation. Arthurs and Moiseiwitsch have calculated values for a parameter S from which the K-ionization cross section σ can be obtained for electron kinetic energies T_1 extending

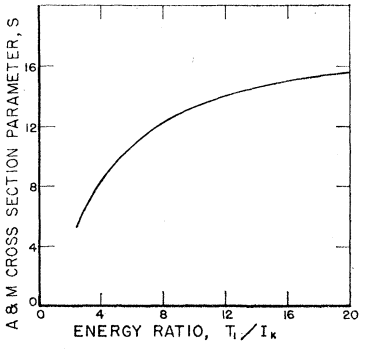

FIG. 1. Dependence of the parameter S on the ratio of the initial electron kinetic energy T_1 to the K-shell binding energy I_K. The parameter S is obtained from the calculations of Arthurs and Moiseiwitsch (Ref. 7) and is related to the K-ionization cross section by Eq. (1).

[1] H. Bethe, Ann. Phys. **5**, 325 (1930).
[2] H. S. W. Massey and C. B. O. Mohr, Proc. Roy. Soc. (London) **A140**, 613 (1933).
[3] D. Graf Soden, Ann. Physik **19**, 409 (1934).
[4] H. S. W. Massey and E. H. S. Burhop, Phys. Rev. **48**, 468 (1935).
[5] W. Wetzel, Phys. Rev. **44**, 25 (1933).
[6] E. H. S. Burhop, Proc. Cambridge Phil. Soc. **36**, 43 (1940).
[7] A. M. Arthurs and B. L. Moiseiwitsch, Proc. Roy. Soc. (London) **A247**, 550 (1958).
[8] H. S. Perlman, Proc. Phys. Soc. (London) **76**, 623 (1960).

from 2.5 to 20 times the K-shell binding energy I_K. The parameter S, which is plotted in Fig. 1 as a function of the energy ratio T_1/I_K, is defined as

$$S=\left[\frac{(137Z_e)^2(E_1^2-1)}{\pi(a_0E_1)^2}\right]\sigma, \quad (1)$$

where Z_e is the effective nuclear charge for the K shell, and is equal to $(Z-0.3)$ where Z is the atomic number, E_1 is the total electron energy in m_0c^2 units, and a_0 is the hydrogen Bohr radius equal to 0.53×10^{-8} cm. Perlman[8] has evaluated the K-ionization cross section for nickel and mercury for electron kinetic energies extending from 3 to 18 times the K-shell binding energy.

Previous experimental investigations of K ionization by electrons have been confined to electron kinetic energies less than 180 keV, and to atomic numbers less than 50. In the relativistic region of electron energies ($T_1\gg1$ keV), K-ionization cross sections were determined for nickel and silver from measurements with an ionization chamber of the intensity of the K x rays emitted when the electrons bombard the target atoms. For nickel, Pockman, Webster, Kirkpatrick, and Harworth[9] measured the relative dependence of the cross section on the electron energy and normalized their data with the absolute cross section at 70 keV measured by Smick and Kirkpatrick.[10] For silver, Webster, Hansen, and Duveneck[11] measured the relative dependence of the cross section on the electron energy, and their data were normalized at 70 keV with the absolute cross section measured by Clark.[12]

The results of the above studies of the K-ionization cross sections for nickel and silver in the energy region from approximately 10 to 180 keV are summarized in Fig. 2. These results give the following information: For nickel, the experimental cross sections of Pockman *et al.*[9] agree within a few percent with the theoretical cross sections predicted by Arthurs and Moiseiwitsch.[7] For silver, the experimental cross sections of Clark[12] and of Webster *et al.*[11] are approximately 20% less than the theoretical cross sections predicted by Arthurs and Moiseiwitsch.[7] In addition Burhop's nonrelativistic cross sections[6] are also approximately 20% less than the relativistic cross sections of Arthurs and Moiseiwitsch,[7] and the disagreement increases as the incident electron energy and the K-shell binding energy increases.

As a result of the disagreement for silver shown in Fig. 2, there is considerable uncertainty about the accuracy of the K-ionization cross sections predicted for relativistic electron energies by the calculations of Arthurs and Moiseiwitsch[7] and by the more recent completely relativistic calculations of Perlman.[8] More data are needed for the higher atomic numbers and energies. The present measurements are carried out to determine the K-ionization cross sections for the higher atomic numbers of 50 (tin, with the K-shell binding energy I_K equal to approximately 29.2 keV)[13] and 79 (gold, with the K-shell binding energy I_K equal to approximately 80.7 keV),[13] and for electron kinetic energies of 50, 100, 200, and 500 keV.

2. EXPERIMENTAL METHOD

In these measurements, thin targets of tin and gold are bombarded by a beam of monoenergetic electrons at 50, 100, 200, and 500 keV from the NBS constant potential accelerator. The K x rays emitted from each target are detected and analyzed with a sodium iodide scintillation spectrometer[14] at a given angle with respect to the incident electron direction. A few samples of the pulse-height distribution produced in the scintillator by the K x rays, approximately 25 keV [13] for tin and 70 keV [13] for gold, are shown in Fig. 3 for the energy region below 100 keV.

The experimental cross section σ for the ionization of the atomic K-shell electrons, is determined from the following equation:

$$\sigma=4\pi N_K/mn\Delta\Omega\epsilon\omega_K. \quad (2)$$

Definitions and measurements pertaining to the quantities in Eq. (2) are given below.

The quantity N_K is the number of K x rays detected by the scintillation spectrometer for a given target,

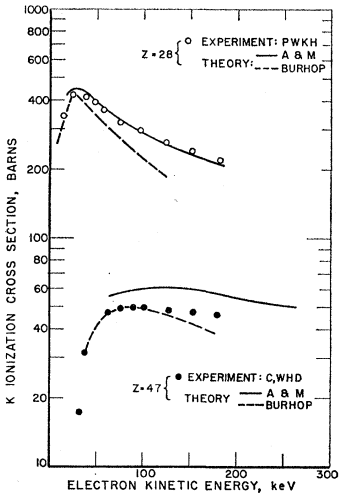

FIG. 2. Dependence of the K-ionization cross section for nickel and silver on the initial electron kinetic energy. The circles give the experimental cross sections of Pockman *et al.* (Ref. 9) (PWKH) for nickel and of Clark *et al.* (Refs. 11 and 12) (C, WHD) for silver. The solid and broken lines give the theoretical cross sections respectively of Arthurs and Moiseiwitsch (Ref. 7) (A & M) and of Burhop (Ref. 6).

[9] L. T. Pockman, D. L. Webster, P. Kirkpatrick, and K. Harworth, Phys. Rev. **71**, 330 (1947).

[10] A. E. Smick and P. Kirkpatrick, Phys. Rev. **67**, 153 (1945).

[11] D. L. Webster, W. W. Hansen, and F. B. Duveneck, Phys. Rev. **43**, 851 (1933).

[12] J. C. Clark, Phys. Rev. **48**, 30 (1935).

[13] S. Fine and C. F. Hendee, Nucleonics **13**, 36 (1955).

[14] Descriptive details about this spectrometer are given by J. W. Motz and R. C. Placious, Phys. Rev. **109**, 235 (1958).

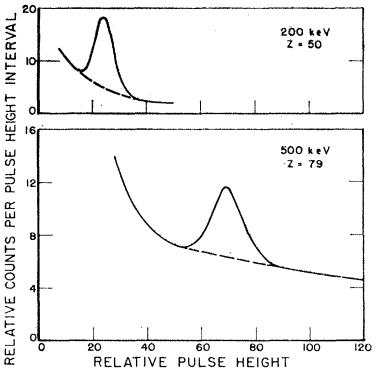

FIG. 3. Pulse-height distributions obtained for the K x rays (approximately 25 keV for tin and 70 keV for gold) with the scintillation spectrometer at 70 deg to the direction of the incident electrons at energies of 200 and 500 keV.

electron energy, and total electron charge incident on the target. For a given case, N_K is equal to the area under the line shape for the K x rays, which is shown in Fig. 3, where the dashed base line is extrapolated along the continuous curve which arises from the background bremsstrahlung radiation. Equation (2) requires isotropic emission for the K x rays, and as a check, measurements of N_K for each target and electron energy were made with the detector at angles of 70 and 110 deg with respect to the direction of the incident electron beam. The results showed that within the experimental errors, the cross sections calculated from Eq. (2) are independent of the angle selected for the K x-ray detector.

The quantity n is determined from the effective target thickness and is equal to the number of target atoms per cm^2 normal to the beam direction. For each electron energy and atomic number, measurements were made with two different effective target thicknesses, approximately 70 and 100 μg/cm^2 for gold and approximately 90 and 185 μg/cm^2 for tin.

The quantity $\Delta\Omega$ is the solid angle subtended by the area of the collimator opening to the scintillation detector. For these measurements, $\Delta\Omega$ is equal to 4.5×10^{-4} sr from a point at the center of the target. Furthermore, photon penetration and scattering effects through the collimator edges are estimated to contribute less than 2% to the measured intensity of the K x rays.

The quantity ϵ is the efficiency of the scintillation spectrometer for the detection of the K x rays emitted from the target into the solid angle $\Delta\Omega$ subtended by the collimator opening of the detector. This efficiency includes corrections for the photon absorption in the windows of the target chamber and the scintillator, for the escape of the iodine K x rays[15] in the sodium iodide scintillator. For the crystal-collimator-target geometry employed in these measurements, the efficiency was estimated to be equal to 0.89 and 0.98 for the K x rays from gold and tin, respectively.

[15] T. B. Novey, Phys. Rev. 89, 672 (1953).

The quantity m is equal to the number of electrons incident on the target, and is determined from the total electron charge collected by a Faraday cup and measured with a current integrator. The quantity ω_K is the probability that a K x ray will be emitted from the atom when an electron is removed from the K shell (K-shell fluorescent yield), and is equal to 0.84 and 0.95 for tin and gold, respectively, from the data of Wapstra *et al.*[16]

Estimates of the systematic errors involved in the measurements of the above quantities may be summarized for most of the data as follows: (1) $\pm5\%$ for N_K, (2) $\pm5\%$ for n, (3) $\pm3\%$ for $\Delta\Omega$, (4) $\pm2\%$ for ϵ, (5) $\pm2\%$ for m, and (6) $\pm1\%$ for ω_K. On the basis of the above estimates, the experimental cross sections are expected to have an accuracy to within approximately 15% of the exact values.

3. RESULTS AND DISCUSSION

Experimental values of the cross section for the atomic K-shell ionization of tin and gold are shown by the open and closed circles, respectively, in Fig. 4. The values given by the error limits are obtained with the inclusion of the best estimates of the systematic errors listed in Sec. 2. Also, the solid lines in Fig. 4 give the theoretical K-ionization cross sections for tin and gold predicted by the calculations of Arthurs and Moiseiwitsch.[7] The broken lines in Fig. 4 give the theoretical K-ionization cross sections for mercury, and are separately identified with the calculations of Arthurs and Moiseiwitsch,[7] Perlman,[8] and Burhop.[6] Unfortunately Perlman and Burhop do not give results for gold, but the differences in the theoretical cross sections for gold and mercury can be expected to be small enough (as indicated by the gold and mercury results obtained from the calculations of Arthurs and Moiseiwitsch) to permit at least a qualitative comparison with the experimental results.

The results in Fig. 4 indicate that the experimental cross sections for tin and gold obtained in the present measurements show good agreement with the theoretical values predicted by the calculations of Arthurs and Moiseiwitsch. For tin, the agreement is especially good over the energy region from 50 to 500 keV, and for gold, the agreement is better than 20% at 500 keV, and is uncertain in the energy region below 200 keV, which is close to the K-ionization threshold for gold. On the other hand, the experimental cross sections for gold are larger by 50% or more than the theoretical cross sections calculated by Perlman and by Burhop.

The important differences in the calculations of (1) Arthurs and Moiseiwitsch, (2) Perlman, and (3) Burhop can be summarized as follows: (1) Arthurs and Moiseiwitsch give a relativistic treatment for the

[16] A. H. Wapstra, G. I. Nijgh, and R. Van Lieshout, *Nuclear Spectroscopy Tables* (North-Holland Publishing Company, Amsterdam, 1959), p. 82.

projectile electron and a nonrelativistic treatment for the atomic electron. For the projectile electron, the initial and final states are represented by relativistic free-particle wave functions. For the atomic electron, the initial and final states are represented by a nonrelativistic hydrogenic wave function and a nonrelativistic Coulomb wave function, respectively. In addition, these calculations do not include exchange effects and are expected to be valid for $(Z/137)^2 \ll 1$ and for $q \ll 1$, where Z is the atomic number and q is the momentum (in m_0c units) transferred to the atom. (2) Perlman gives a relativistic treatment for both the projectile and the atomic electrons. For the projectile electron, the initial and final states are represented by relativistic free-particle wave functions. For the atomic electron, the intial and final states are represented by a hydrogenic Dirac wave function and a relativistic Coulomb wave function, respectively. Also, these calculations do not include exchange effects. (3) Finally, Burhop gives a nonrelativistic treatment for both the projectile and atomic electrons. For the projectile electron, the initial and final states are represented by nonrelativistic free-particle wave functions. For the atomic electron, the initial and final states are represented by nonrelativistic hydrogenic wave functions and nonrelativistic Coulomb wave functions, respectively. Also, these calculations do not include exchange effects. From the above comparison, the completely relativistic calculations of Perlman can be expected to give the most accurate cross-section values. For this reason, the large disagreement of Perlman's results with the present measurements is not understood, unless there are errors in the numerical evaluation of Perlman's complicated formulas. It should be noted that Perlman obtains good agreement with Arthurs and Moiseiwitsch for the case of nickel with simplified wave functions.

The predictions of Arthurs and Moiseiwitsch show surprisingly good agreement with the present experimental results in Fig. 4 for tin and gold, and with previous experimental results[9] in Fig. 2 for nickel. Although the experimental curve by Clark *et al.*[11,12] for silver is approximately 20% lower than the theoretical curve, the shapes of both curves show good agreement; this difference in absolute values, which is

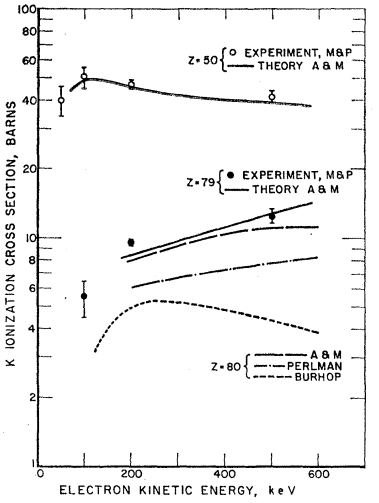

FIG. 4. Dependence of the K-ionization cross section for tin and gold on the initial electron kinetic energy. The circles give the experimental cross sections obtained in the present measurements (M & P). The solid and broken lines are identified separately and give the theoretical cross sections for tin, gold, and mercury, which are obtained from the calculations of Arthurs and Moiseiwitsch (Ref. 7) (A & M), Perlman (Ref. 8) and Burhop (Ref. 6).

contradicted by the present results for tin, may be reasonably attributed to an experimental error in Clark's normalization of the data at 70 keV. The general agreement shown in Figs. 2 and 4 of the calculations of Arthurs and Moiseiwitsch with the experimental results indicates that in the energy region from 50 to 500 keV, the important contribution to the K-ionization cross section for both low- and high-Z atoms comes from collisions involving a small momentum transfer for which relativistic effects for the atomic electron are not important, and for which the relativistic free-particle wave functions are a good approximation for the projectile electron, except for uncertainties in the threshold energy region.

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